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Serial No.....

Society of Certified Management Accountants of Sri Lanka

Technician Stage September 2008 Examination

Examination Date : 4th October 2008
Examination Time : 9.30a.m.- 12.30p.m.

Number of Pages : 10
Number of Questions: 09

Instructions to the Candidates

1. Time allowed is **three (3)** hours
2. Answer **any five(5)** questions
3. Answers should be entirely in the **English** language

<u>Subject</u>	<u>Subject Code</u>
Business Mathematics	(BMT / 503)

Question No. 1 (20 Marks)

- (a) Two whole numbers x and y are such that their sum is 40.
- (i) Express y in terms of x . (1 Mark)
 - (ii) If $S(x)$ denotes the sum of their squares, show that
$$S(x) = 2x^2 - 80x + 1600$$
 (3 Marks)
 - (iii) Sketch the graph of $S(x)$. (4 Marks)
 - (iv) Which two numbers produce the smallest sum of squares? (2 Marks)
- (b) If A is the square matrix given by
- $$A = \begin{bmatrix} 4 & -1 \\ 2 & 1 \end{bmatrix},$$
- (i) Show that $A^2 - 5A + 6I = 0$, where I is the 2×2 unit matrix and 0 is the 2×2 zero matrix. (5 Marks)
 - (ii) Hence find A^{-1} . (5 Marks)
- (Total 20 Marks)**

Question No. 2 (20 Marks)

- (a) The sum of the first n terms of an arithmetic series is given by
- $$S_n = 2n^2 - 3n$$
- (i) Find the n^{th} term of the series. (4 Marks)
 - (ii) Find the common difference of the series. (3 Marks)
- (b) The first term of a geometric series is 1 and its common ratio is 3:2.
- (i) Write an expression for finding the sum of the first n terms S_n of the series. (5 Marks)
 - (ii) Find the smallest value of n such that $S_n > 1000$. (8 Marks)
- (Total 20 Marks)**

Question No. 3 (20 Marks)

(a) Solve the quadratic equation.

$$(2x-1)(x+2)=12$$

(5

Marks)

(b) By the use of factors find the following product and express the answer as a trinomial

$$(a+1)^2(a-1)^2(a^2+1)^2$$

(5 Marks)

(c) Simplify and express (i) and (ii) with positive exponents.

$$(i) \left(\frac{x y^{-1}}{x^{\frac{1}{2}} y^{\frac{-3}{2}}} \right)^{-2}$$

(5 Marks)

$$(ii) (a^x - b^y)(a^{-x} + b^{-y})$$

(5 Marks)

(Total 20 Marks)

Question No. 4 (20 Marks)

For a particular random experiment, the following probabilities were observed.

$$P(A|B)=0.6, P(A^1|B^1)=0.1, P(B)=0.4,$$

Where A^1 and B^1 denote the complements of the events A and B .

(a) Find the following probabilities.

$$(i) P(A \cap B)$$

(1 Mark)

$$(ii) P(A \cap B^1)$$

(1 Mark)

$$(iii) P(A^1 \cap B)$$

(1 Mark)

$$(iv) P(A)$$

(1 Mark)

$$(v) P(B|A)$$

(1 Mark)

$$(vi) P(A^1 \cup B^1)$$

(3 Marks)

(b) The random variable x of the above experiment takes value 4 if both A and B occur, 3 if A occurs and not B , 2 if B occurs and not A , and 1 if neither A nor B occurs.

(i) Specify the probability distribution of x .

(4 Marks)

(ii) Find the mean of x .

(3 Marks)

(iii) Find the variance of x .

(5 Marks)

(Total 20 Marks)

Question No. 5 (20 Marks)

The annual incomes in billions of rupees of 120 companies for the year 2007 are as follows:

Income	Number of companies
1 – 50	45
51 – 100	30
101 – 200	20
201 – 300	15
301 – 500	10

- (a) Draw a histogram to represent these data. **(4 Marks)**
- (b) Estimate the median and inter quartile range of annual incomes. **(6 Marks)**
- (c) Estimate the mean and standard deviation of these data. **(9 Marks)**
- (d) State whether a statistician should use the median and the inter quartile range or the mean and the standard deviation to compare annual incomes. Give reasons for your answer. **(1 Mark)**
(Total 20 Marks)

Question No. 6 (20 Marks)

- (a) A random sample of 9 items has a mean of 56 and standard deviation 3.3. Use the t -distribution to determine whether it could have been drawn from a normal population whose mean is 52. Test at 2% level. **(6 Marks)**
- (b) The following table shows the number of patients suffering from certain disease, between the years 1575 and 2006.

Number of patients in a year	0	1	2	3	4	> 4
Number of years	223	142	48	14	4	0

- (i) Find the mean of the distribution. **(2 Marks)**
- (ii) Fit the appropriate Poisson distribution. **(6 Marks)**
- (iii) Test the goodness of fit using χ^2 distribution at 5% level. **(6 Marks)**
(Total 20 Marks)

Question No. 7 (20 Marks)

Ten students took tests in mathematics and statistics. The number of students and the marks obtained by them are given in the table below, where x represents the mathematics and y the statistics marks.

		Student									
		A	B	C	D	E	F	G	H	I	J
Mark	x	54	60	60	62	72	80	81	84	90	96
	y	62	72	82	80	78	84	88	90	96	96

- (a) Draw a scatter diagram to illustrate these data. **(6 Marks)**
- (b) Find the equation of the regression line in the form $y = a + bx$, where a and b are constants. **(12 Marks)**

- (c) An eleventh student was absent for the statistics test. However he sat for the mathematics test and scored 70. Using this model, estimate the mark he would have scored in the statistics test. **(2 Marks)**

(Total 20 Marks)

Question No. 8 (20 Marks)

- (a) The set of constraints for a linear programming problem are given by

$$2x + 5y \leq 50, \quad 4x + y \leq 28, \quad x \geq 0 \quad \text{and} \quad y \geq 0.$$

- (i) Sketch the feasible region. **(6 Marks)**
 (ii) Maximize the objective function $z = 3x + 4y$, subject to these constraints. **(4 Marks)**

- (b) The demand equation for a certain product is given by

$$p = 500 - 0.5e^{0.004x},$$

Where p is the price and x is the number of items produced.

- (i) Find the price for the demand of 100 items. **(3 Marks)**
 (ii) Find the demand for a price of Rs.300/-. **(7 Marks)**

(Total 20 Marks)

Question No. 9 (20 Marks)

- (a) For a certain item the demand per year is 2500 units, produce cost is Rs.110, the fixed ordering cost is Rs.16, and the carrying cost rate is 0.20.

- (i) Compute the economic order quantity, Q . **(2 Marks)**
 (ii) Compute minimum total annual cost. **(3 Marks)**
 (iii) Round Q to the nearest whole number. Compute the percentage increase in the total cost increased due to rounding the economic order quantity, Q . **(3 Marks)**

- (b) The activities, predecessors and time duration (in weeks) of a project are given below.

Activity	A	B	C	D	E	F	G	H	I
Predecessors	-	A	-	B,C	D	D	E	F,G	E
Time duration	4	2	2	3	7	4	3	2	2

- (i) Construct a project network. **(4 Marks)**
 (ii) Compute ES, EF, LS, LF and slack times. **(7 Marks)**
 (iii) Identify the critical path. **(1 Mark)**

(Total 20 Marks)

List of Formulae

1. Sum to n terms of an arithmetic progression:

$$S_n = \frac{a(r^n - 1)}{r - 1}$$

Where, r = common ratio
 a = first term
 n = number of terms
 S_n = sum to n terms of the series

2. Probability:

$$(i) \quad p(C \cup D) = p(C) + p(D) - p(C \cap D)$$

$$(ii) \quad p(C/D) = \frac{p(C \cap D)}{p(D)},$$

Where C and D are two events of the same sample space.

3. Variance of a population:

$$\sigma^2 = E(X^2) - [E(X)]^2$$

Where, $E(X)$ = the expected value of the random variable X
 $E(X^2)$ = the expected value of X^2

4. Median of a grouped data set :

$$\text{Median} = \left(\frac{\frac{n}{2} - f_a}{f_b} \right) i + B_L,$$

Where, B_L = lower boundary of the median class
 f_a = cumulative frequency of the class preceding the median class
 f_b = number of observations in the median class
 i = width of the median class
 n = total number of observations in the distribution

5. Variance of a frequency distribution :

$$\sigma^2 = \frac{\sum f_i x_i^2 - n \bar{x}^2}{n - 1}$$

Where, x_i = i^{th} measurement
 f_i = frequency of the i^{th} class

\bar{x} = mean of the distribution
 n = total number of observations

6. **t - statistic :**

$$t = \frac{\bar{x} - \mu}{S / \sqrt{n}}$$

Where, \bar{x} = mean of the sample
 μ = mean of the population
 S = standard deviation of the sample
 n = sample size

7. **χ^2 - statistic :**

$$\chi^2 = \sum \frac{(O_i - E_i)^2}{E_i}$$

Where, O_i = observed frequency of the i^{th} class
 E_i = theoretical frequency of the i^{th} class

8. **Poisson distribution :**

$$P(X = x) = \frac{e^{-\lambda} \lambda^x}{x!}$$

Where, λ = Poisson mean
 x = Poisson random variable

9. **Equation of the Regression Line:**

$$y = a + bx$$

$$\text{Where, } b = \frac{\sum xy - \frac{\sum x \sum y}{n}}{\sum x^2 - \frac{(\sum x)^2}{n}}, \quad a = \frac{\sum y - b \sum x}{n}$$

10. **Economic Order Quantity:**

$$Q = \sqrt{\frac{2DF}{cr}}$$

Where, D = annual demand
 F = ordering cost per order
 C = product cost per unit
 r = carrying cost rate

$$\text{Total annual cost} = F(D/Q) + cr(Q/2)$$

Present value table

Present value of 1.00 unit of currency, that is $(1 + r)^{-n}$ where r = interest rate; n = number of periods until payment or receipt.

Periods (n)	Interest rates (r)									
	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%
1	0.990	0.980	0.971	0.962	0.952	0.943	0.935	0.926	0.917	0.909
2	0.980	0.961	0.943	0.925	0.907	0.890	0.873	0.857	0.842	0.826
3	0.971	0.942	0.915	0.889	0.864	0.840	0.816	0.794	0.772	0.751
4	0.961	0.924	0.888	0.855	0.823	0.792	0.763	0.735	0.708	0.683
5	0.951	0.906	0.863	0.822	0.784	0.747	0.713	0.681	0.650	0.621
6	0.942	0.888	0.837	0.790	0.746	0.705	0.666	0.630	0.596	0.564
7	0.933	0.871	0.813	0.760	0.711	0.665	0.623	0.583	0.547	0.513
8	0.923	0.853	0.789	0.731	0.677	0.627	0.582	0.540	0.502	0.467
9	0.914	0.837	0.766	0.703	0.645	0.592	0.544	0.500	0.460	0.424
10	0.905	0.820	0.744	0.676	0.614	0.558	0.508	0.463	0.422	0.386
11	0.896	0.804	0.722	0.650	0.585	0.527	0.475	0.429	0.388	0.350
12	0.887	0.788	0.701	0.625	0.557	0.497	0.444	0.397	0.356	0.319
13	0.879	0.773	0.681	0.601	0.530	0.469	0.415	0.368	0.326	0.290
14	0.870	0.758	0.661	0.577	0.505	0.442	0.388	0.340	0.299	0.263
15	0.861	0.743	0.642	0.555	0.481	0.417	0.362	0.315	0.275	0.239
16	0.853	0.728	0.623	0.534	0.458	0.394	0.339	0.292	0.252	0.218
17	0.844	0.714	0.605	0.513	0.436	0.371	0.317	0.270	0.231	0.198
18	0.836	0.700	0.587	0.494	0.416	0.350	0.296	0.250	0.212	0.180
19	0.828	0.686	0.570	0.475	0.396	0.331	0.277	0.232	0.194	0.164
20	0.820	0.673	0.554	0.456	0.377	0.312	0.258	0.215	0.178	0.149

Periods (n)	Interest rates (r)									
	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%
1	0.901	0.893	0.885	0.877	0.870	0.862	0.855	0.847	0.840	0.833
2	0.812	0.797	0.783	0.769	0.756	0.743	0.731	0.718	0.706	0.694
3	0.731	0.712	0.693	0.675	0.658	0.641	0.624	0.609	0.593	0.579
4	0.659	0.636	0.613	0.592	0.572	0.552	0.534	0.516	0.499	0.482
5	0.593	0.567	0.543	0.519	0.497	0.476	0.456	0.437	0.419	0.402
6	0.535	0.507	0.480	0.456	0.432	0.410	0.390	0.370	0.352	0.335
7	0.482	0.452	0.425	0.400	0.376	0.354	0.333	0.314	0.296	0.279
8	0.434	0.404	0.376	0.351	0.327	0.305	0.285	0.266	0.249	0.233
9	0.391	0.361	0.333	0.308	0.284	0.263	0.243	0.225	0.209	0.194
10	0.352	0.322	0.295	0.270	0.247	0.227	0.208	0.191	0.176	0.162
11	0.317	0.287	0.261	0.237	0.215	0.195	0.178	0.162	0.148	0.135
12	0.286	0.257	0.231	0.208	0.187	0.168	0.152	0.137	0.124	0.112
13	0.258	0.229	0.204	0.182	0.163	0.145	0.130	0.116	0.104	0.093
14	0.232	0.205	0.181	0.160	0.141	0.125	0.111	0.099	0.088	0.078
15	0.209	0.183	0.160	0.140	0.123	0.108	0.095	0.084	0.079	0.065
16	0.188	0.163	0.141	0.123	0.107	0.093	0.081	0.071	0.062	0.054
17	0.170	0.146	0.125	0.108	0.093	0.080	0.069	0.060	0.052	0.045
18	0.153	0.130	0.111	0.095	0.081	0.069	0.059	0.051	0.044	0.038
19	0.138	0.116	0.098	0.083	0.070	0.060	0.051	0.043	0.037	0.031
20	0.124	0.104	0.087	0.073	0.061	0.051	0.043	0.037	0.031	0.026

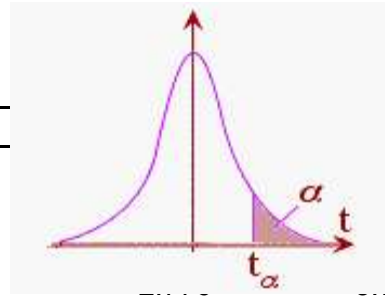
Cumulative present value of 1.00 unit of currency per annum, Receivable or

Payable at the end of each year for n years $\left[\frac{1-(1+r)^{-n}}{r} \right]$

Periods (n)	Interest rates (r)									
	1%	2%	3%	4%	5%	6%	7%	8%	9%	10%
1	0.990	0.980	0.971	0.962	0.952	0.943	0.935	0.926	0.917	0.909
2	1.970	1.942	1.913	1.886	1.859	1.833	1.808	1.783	1.759	1.736
3	2.941	2.884	2.829	2.775	2.723	2.673	2.624	2.577	2.531	2.487
4	3.902	3.808	3.717	3.630	3.546	3.465	3.387	3.312	3.240	3.170
5	4.853	4.713	4.580	4.452	4.329	4.212	4.100	3.993	3.890	3.791
6	5.795	5.601	5.417	5.242	5.076	4.917	4.767	4.623	4.486	4.355
7	6.728	6.472	6.230	6.002	5.786	5.582	5.389	5.206	5.033	4.868
8	7.652	7.325	7.020	6.733	6.463	6.210	5.971	5.747	5.535	5.335
9	8.566	8.162	7.786	7.435	7.108	6.802	6.515	6.247	5.995	5.759
10	9.471	8.983	8.530	8.111	7.722	7.360	7.024	6.710	6.418	6.145
11	10.368	9.787	9.253	8.760	8.306	7.887	7.499	7.139	6.805	6.495
12	11.255	10.575	9.954	9.385	8.863	8.384	7.943	7.536	7.161	6.814
13	12.134	11.348	10.635	9.986	9.394	8.853	8.358	7.904	7.487	7.103
14	13.004	12.106	11.296	10.563	9.899	9.295	8.745	8.244	7.786	7.367
15	13.865	12.849	11.938	11.118	10.380	9.712	9.108	8.559	8.061	7.606
16	14.718	13.578	12.561	11.652	10.838	10.106	9.447	8.851	8.313	7.824
17	15.562	14.292	13.166	12.166	11.274	10.477	9.763	9.122	8.544	8.022
18	16.398	14.992	13.754	12.659	11.690	10.828	10.059	9.372	8.756	8.201
19	17.226	15.679	14.324	13.134	12.085	11.158	10.336	9.604	8.950	8.365
20	18.046	16.351	14.878	13.590	12.462	11.470	10.594	9.818	9.129	8.514

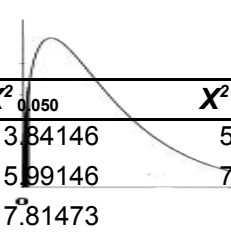
Periods (n)	Interest rates (r)									
	11%	12%	13%	14%	15%	16%	17%	18%	19%	20%
1	0.901	0.893	0.885	0.877	0.870	0.862	0.855	0.847	0.840	0.833
2	1.713	1.690	1.668	1.647	1.626	1.605	1.585	1.566	1.547	1.528
3	2.444	2.402	2.361	2.322	2.283	2.246	2.210	2.174	2.140	2.106
4	3.102	3.037	2.974	2.914	2.855	2.798	2.743	2.690	2.639	2.589
5	3.696	3.605	3.517	3.433	3.352	3.274	3.199	3.127	3.058	2.991
6	4.231	4.111	3.998	3.889	3.784	3.685	3.589	3.498	3.410	3.326
7	4.712	4.564	4.423	4.288	4.160	4.039	3.922	3.812	3.706	3.605
8	5.146	4.968	4.799	4.639	4.487	4.344	4.207	4.078	3.954	3.837
9	5.537	5.328	5.132	4.946	4.772	4.607	4.451	4.303	4.163	4.031
10	5.889	5.650	5.426	5.216	5.019	4.833	4.659	4.494	4.339	4.192
11	6.207	5.938	5.687	5.453	5.234	5.029	4.836	4.656	4.486	4.327
12	6.492	6.194	5.918	5.660	5.421	5.197	4.988	4.793	4.611	4.439
13	6.750	6.424	6.122	5.842	5.583	5.342	5.118	4.910	4.715	4.533
14	6.982	6.628	6.302	6.002	5.724	5.468	5.229	5.008	4.802	4.611
15	7.191	6.811	6.462	6.142	5.847	5.575	5.324	5.092	4.876	4.675
16	7.379	6.974	6.604	6.265	5.954	5.668	5.405	5.162	4.938	4.730
17	7.549	7.120	6.729	6.373	6.047	5.749	5.475	5.222	4.990	4.775
18	7.702	7.250	6.840	6.467	6.128	5.818	5.534	5.273	5.033	4.812
19	7.839	7.366	6.938	6.550	6.198	5.877	5.584	5.316	5.070	4.843
20	7.963	7.469	7.025	6.623	6.259	5.929	5.628	5.353	5.101	4.870

t- Distribution table



d.f.	$t_{.100}$	$t_{.050}$		$t_{.010}$	d.f.
1	3.078	6.314			1
2	1.886	2.920			2
3	1.638	2.353			3
4	1.533	2.132			4
5	1.476	2.015	2.571	3.365	5
6	1.440	1.943	2.447	3.143	6
7	1.415	1.895	2.365	2.998	7
8	1.397	1.860	2.306	2.896	8
9	1.383	1.833	2.262	2.821	9
10	1.372	1.812	2.228	2.764	10
11	1.363	1.796	2.201	2.718	11
12	1.356	1.782	2.179	2.681	12
13	1.350	1.771	2.160	2.650	13
14	1.345	1.761	2.145	2.624	14
15	1.341	1.753	2.131	2.602	15
16	1.337	1.746	2.120	2.583	16
17	1.333	1.740	2.110	2.567	17
18	1.330	1.734	2.101	2.552	18
19	1.328	1.729	2.093	2.539	19
20	1.325	1.725	2.086	2.528	20
21	1.323	1.721	2.080	2.518	21
22	1.321	1.717	2.074	2.508	22
23	1.319	1.714	2.069	2.500	23
24	1.318	1.711	2.064	2.492	24
25	1.316	1.708	2.060	2.485	25
26	1.315	1.706	2.056	2.479	26
27	1.314	1.703	2.052	2.473	27
28	1.313	1.701	2.048	2.467	28
29	1.311	1.699	2.045	2.462	29
Inf.	1.282	1.645	1.960	2.326	Inf.

X^2 – distribution table



d.f.	$X^2_{0.100}$	$X^2_{0.050}$	$X^2_{0.025}$	$X^2_{0.010}$	$X^2_{0.005}$	d.f.
1	2.70554	3.84146	5.02389	6.6349	7.87944	1
2	4.60517	5.99146	7.37776	9.21034	10.59663	2
3	6.25139	7.81473	9.3484	11.34487	12.83816	3
4	7.77944	9.48773	11.14329	13.2767	14.86026	4
5	9.23635	11.0705	12.8325	15.08627	16.7496	5
6	10.64464	12.59159	14.44938	16.81189	18.54758	6
7	12.01704	14.06714	16.01276	18.47531	20.27774	7
8	13.36157	15.50731	17.53455	20.09024	21.95495	8
9	14.68366	16.91898	19.02277	21.66599	23.58935	9
10	15.98718	18.30704	20.48318	23.20925	25.18818	10
11	17.27501	19.67514	21.92005	24.72497	26.75685	11
12	18.54935	21.02607	23.33666	26.21697	28.29952	12
13	19.81193	22.36203	24.7356	27.68825	29.81947	13
14	21.06414	23.68479	26.11895	29.14124	31.31935	14
15	22.30713	24.99579	27.48839	30.57791	32.80132	15
16	23.54183	26.29623	28.84535	31.99993	34.26719	16
17	24.76904	27.58711	30.19101	33.40866	35.71847	17
18	25.98942	28.8693	31.52638	34.80531	37.15645	18
19	27.20357	30.14353	32.85233	36.19087	38.58226	19
20	28.41198	31.41043	34.16961	37.56623	39.99685	20
21	29.61509	32.67057	35.47888	38.93217	41.40106	21
22	30.81328	33.92444	36.78071	40.28936	42.79565	22
23	32.0069	35.17246	38.07563	41.6384	44.18128	23
24	33.19624	36.41503	39.36408	42.97982	45.55851	24
25	34.38159	37.65248	40.64647	44.3141	46.92789	25
26	35.56317	38.88514	41.92317	45.64168	48.28988	26
27	36.74122	40.11327	43.19451	46.96294	49.64492	27
28	37.91592	41.33714	44.46079	48.27824	50.99338	28
29	39.08747	42.55697	45.72229	49.58788	52.33562	29
30	40.25602	43.77297	46.97924	50.89218	53.67196	30
40	51.8050	55.7585	59.3417	63.6907	66.7659	40
50	63.1671	67.5048	71.4202	76.1539	79.4900	50
60	74.3970	79.0819	83.2976	88.3794	91.9517	60
70	85.5271	90.5312	95.0231	100.425	104.215	70
80	96.5782	101.145	106.629	112.329	116.321	80
90	107.565	113.145	118.136	124.116	128.299	90
100	118.498	124.342	129.561	135.807	140.169	100

End of Question Paper